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A Possible Explanation of the Sun-spot Period.

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This paper is an attempt to obtain a gravitational cause for the Sun-spot period on somewhat different lines from those which have hitherto been followed.*

The possibility that the period is in some way connected with the positions of the planets has been frequently suggested. Carrington, in his well known work, has given a comparison of the curve formed from the Sun-spot numbers with one having the same period as that of *Jupiter* round the Sun. Balfour Stewart (*Trans. R. S. Edin.*, vol. xxiii.) examined the influence of *Jupiter* and the inner planets. Birkeland (*Vidensk. Skrifter*, 1899, Christiania) has done a similar piece of work on a much more extended scale. These investigations all appear to have produced a negative result as far as the main period is concerned, the chief reason being that the only long-period planet considered, *i.e. Jupiter*, has a period of 11·86 years, so that the examination ultimately reduces to an attempt to make the *Jupiter* curve agree with the Sun-spot curve. It will be seen below that the main difference of the method followed here from that of previous investigators, is the inclusion of an action due to *Saturn* combined with that of *Jupiter*, with possible minor effects due to the inner planets, instead of considering *Jupiter* and the inner planets only. If we compound two periodic curves of nearly equal amplitudes and

* Part of the paper has been re-written since it was first presented to the Society, owing to the valuable criticisms of the referees. My attention was also called by them to the memoir of M. Birkeland, which has appeared during the present year.

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periods not very different, an *apparent* period may result different from the component periods when the series does not extend over a very great number of maxima.

If we examine the tide-raising forces on the Sun due to the planets, we obtain the following relative numbers :—

By *Mercury*	8·6	By *Jupiter*	2·24
,, *Venus*	2·06	,, *Saturn*	·11
,, The Earth	1·00	,, *Uranus*	·002
,, *Mars*	·03	,, *Neptune*	·0006

The mean values of the distances of the planets are used, taking the mean distance and mass of the Earth as units. The mass of *Mercury* is assumed to be $\frac{1}{20}$ that of the Earth ; any probable change in this value will not affect the argument of this paper.

It is at once seen that the inner planets, with the exception of *Mars*, all produce considerable relative effects, but as their periods are very short, it will be assumed that they can be neglected in the search for long-period effects. As *Uranus* and *Neptune* can hardly be supposed, from the numbers given above, to have any influence, *Jupiter* and *Saturn* alone remain.

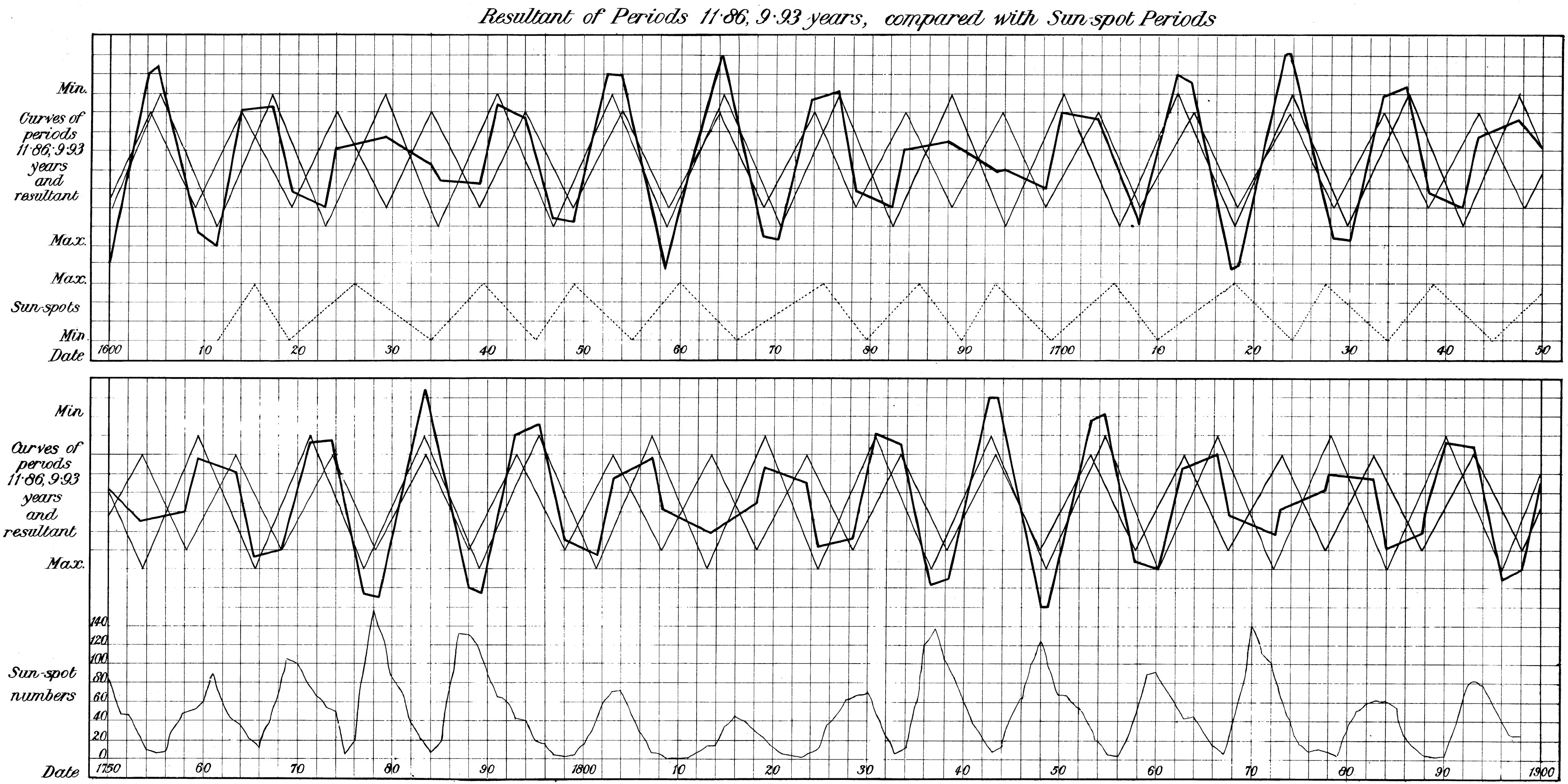
Now *Jupiter* alone, if we neglect the eccentricity of its orbit, cannot in general produce any long-period term, for the period of rotation of the Sun is about twenty-five days, and the wave produced by *Jupiter* travels round the Sun, relatively to the Sun's surface, in very nearly the same period. The eccentricity of the orbit of *Jupiter* is however about $\frac{1}{20}$, and the tide-raising force, 2·24, due to *Jupiter*, will vary by $\pm$·33 as the planet moves in its orbit. This may be considered as a wave superposed on the main *Jupiter* wave, and its period will be that of *Jupiter*, since the motion of the perihelion can be neglected.

Again, *Saturn*, in its motion round the Sun, raises a wave with a force of magnitude ·11. Whenever this wave crosses the main *Jupiter* wave, the latter will have its height increased, and there will be a corresponding diminution when the waves are in quadrature with one another. The period of the wave relative to the main *Jupiter* wave will be that of *Saturn* relative to *Jupiter* round the Sun, that is, 19·86 years. But as a tide-raising force produces equal waves on opposite sides of the Sun, the intervals between coincidences will be just half of this, that is 9·93 years.

Hence the main *Jupiter* wave is periodically altered by two causes :—

	Period.	Mag. of force.
By Jupiter's eccentricity	11·86 years	·33
By the motion of Saturn	9·93 ,,	·11

The two forces are in the ratio 3 : 1, but these numbers cannot contribute anything to the argument further than that they are of the same order of magnitude, even if that is necessary. For



Resultant of Periods 11·86, 9·93 years, compared with Sun-spot Periods
Min.
Curves of periods 11·86, 9·93 years and resultant
Max.
Max.
Sun-spots
Min
Date
1600
10
20
30
40
50
60
70
80
90
1700
10
20
30
40
50
Min
Curves of periods 11·86, 9·93 years and resultant
Max.
140
120
100
80
60
40
20
0
Sun-spot numbers
Date
1750
60
70
80
90
1800
10
20
30
40
50
60
70
80
90
1900

the question under consideration is not the periodic change in the wave height, but a phenomenon here supposed to be due to this change: namely, increase of activity on the Sun's surface due to the relative positions of the waves which in reality form long-period "spring" tides. The periods are the only quantities which admit of more or less exact numerical calculation.

Now, the period of activity of the solar surface has generally been estimated at 11·1 years, and as the number of maxima from which it is deduced is not very large, it might easily have been arrived at even if the actual curves be really compounded from two of not very different amplitudes. It must be remembered also that the period is only an average one; the distances between the various maxima actually range from about 8 to 18 years, so that even if a principal cause with a period of 11·1 years had been found, the deviations would have been considerable.

In the course of the 150 years during which numerical estimates of the activity are available, there is some evidence of a much longer periodic change which appears to run from one maximum to the next in about sixty years. This can be shown to be a direct consequence of the two periods. For

$$5 \times 11{\cdot}86 = 59{\cdot}3,$$
$$6 \times 9{\cdot}93 = 59{\cdot}6,$$

the exact period between two coincidences of phase being

$$1 \div \left(\frac{1}{9{\cdot}93} - \frac{1}{11{\cdot}86}\right) = 61 \text{ years.}$$

Thus every 61 years the two waves are in approximately the same positions relatively to the main *Jupiter* wave, and a corresponding change in the Sun-spot cycle may be expected.

A curve (plate 20) has been drawn, showing the effect of the combination of two periodic terms of periods, 11·86 and 9·93 years respectively. In order to simplify the drawing as much as possible, only the maxima and minima of these were plotted, and they were then joined by (thin) straight lines. The minima were placed at equal distances from the maxima, although a more exact agreement might perhaps have been obtained by making the distance from maximum to minimum longer than that from minimum to maximum. The amplitudes must, from the nature of the case, be somewhat arbitrary, judging from the numerical analysis given later on. They were taken in the ratio of 7 : 5, the larger amplitude being used for the *Jupiter* period. The maxima are placed below and the minima above. The heavy line shows the effect of compounding the two curves.

The lower curve represents Wolf's Sun-spot numbers. In the period before 1750—the dotted portion—the magnitudes are not known; the maxima have therefore been drawn with equal ordinates. For the period since 1750, the Sun-spot curves are

plotted from the annual means given by Wolf.* Thus a period of about 290 years is available for comparison.

In the period before 1750, represented on the upper part of the plate, one must regard Wolf's results with considerable doubt. No connected series of observations were then available. A maximum was sometimes inferred from the mention of a big spot on the Sun easily visible to the naked eye; such a spot has occurred during the minimum now in progress, and therefore one can hardly regard the deduction of a maximum from a big spot as very trustworthy. In fact Wolfer, in rediscussing Wolf's results, gets in one part two maxima and one minimum where Wolf had found only one maximum. Again, it is not easy to decide on what principle (*e.g.* by the number of spots visible in a given time, or by the spotted area) the Sun-spot curves should be formed. The hump which is visible in several cases on the descending slope may possibly come out as the real maximum by another method of reckoning. In fact, the curve gives an appearance of definiteness which disappears when the observations are more closely examined. A reference to the plate attached to Mr. Ellis's paper in the *Monthly Notices* for last December will illustrate this last point; the Sun-spot curves are there drawn on a larger scale from the monthly instead of the annual means.

On comparing the theoretical and observed curves, it will be seen—

(*a*) That the maxima agree on the whole fairly well, and better in the later than in the earlier times.

(*b*) That from 1750–1898 there are three minima and two maxima of the 61-year period which agree fairly well with those of the Sun-spot curve.

(*c*) That at all the maxima of the 61-year period there is an almost exact coincidence of the theoretical and observed maxima.

(*d*) That the deviations from coincidence of the maxima generally occur where they might be expected to occur—namely, round the minima of the 61-year period. This is, in particular, the case before 1750.

(*e*) That the principal deviations are the maxima of dates 1615, 1675, 1894; the deviations of dates 1626, 1685, 1750, 1761 being near the minima of the 61-year period. The first two exceptions are probably due to errors in assigning the dates to these maxima of the Sun-spot curves. Wolf says that there is a probable error of about two years in the date of each maximum before 1750; an error of four years in each of these cases is not improbable. The supposition of error is strengthened by the fact that the slope from maximum to minimum is greater

* Wolf's results have been summarised by A. Wolfer in the *Meteorologische Zeitschrift* for June 1892. The results up to 1891, used here, have been obtained from this paper; for the period 1891–8 the numbers were extracted from Wolfer's annual reports.

than that from minimum to maximum—contrary to what happens with the great majority of the 11·1-year periods.

The deviation of the maximum apparently occurring about 1894 is not so easily explained away, as it is in a period when doubt cannot be cast on the observations. In fact, it appears to coincide more nearly with the theoretical minimum than with the maximum. A general shift back of the theoretical curve will not improve matters. The explanation would probably be that there are other causes tending to displace the position of the actual maximum. The prolonged minimum now in progress seems to confirm this. If the theory developed here is correct, the actual minimum should not be reached until 1901 or 1902, if it is attained as early. The next maximum should be a period of great activity, as it will fall on a maximum of the 61-year period, and it should be attained about 1908. The time-distance from the previous maximum would be about 14 years—an event which is not unusual, as a reference to the plate will show.

The rise of the curve about 1870 is seen by a reference to the monthly means to be due to a very large disturbance lasting a comparatively short time.

(*f*) That the maxima of disturbance do not coincide with the positions of conjunction or opposition of *Saturn* and *Jupiter*. These planets were in conjunction and *Jupiter* was at aphelion in 1827, a date about ten years from the minimum of the 61-year period.

A closer agreement between the two curves can hardly be expected, in view of the many arbitrary quantities which enter into their formation. In addition, the actions of *Mercury*, *Venus*, and the Earth may produce effects comparable with those produced by *Jupiter* and *Saturn*; they might well cause considerable deviations of the maxima at any one period, and their effects would only be eliminated by an analysis of the observations extending over a considerable period of time. It therefore seems advisable to examine the numbers, and to see what evidence of the existence of the periods the observations afford.

For this purpose the mean annual values of the Sun-spot numbers given by Wolf were used. The period 1761–1880 is the longest complete cycle which is available; this includes 12 maxima of the 9·93-year period, and 10 of the 11·86-year period. Arranging these in groups of 10, for the former period we obtain the following mean values :—

1	2	3	4	5	6	7	8	9	10
57	47	34	30	27	38	54	65	67	64

The numbers 1, 2 . . . are the parts of a period, and the mean under each is derived from twelve annual means, each of the latter being derived from the twelve monthly numbers

given by Wolf. Although the period is rather less than ten years, the difference is not sufficient to have any material effect in the course of 120 years. These ten means are plotted in fig. 1, the numbers at the side corresponding to Wolf's Sun-spot numbers, and those below corresponding to the above ten parts of a period.

9·93-year Period.

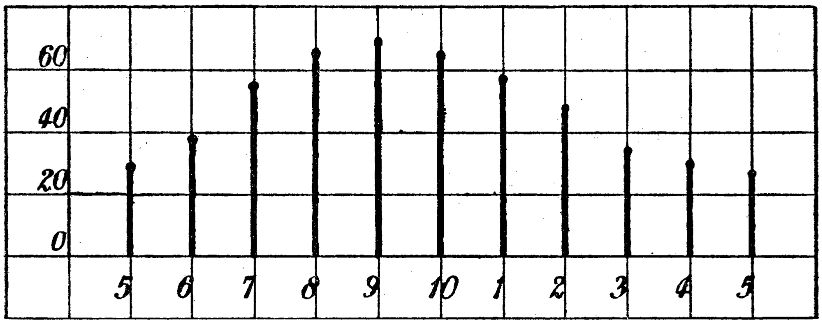


FIG. 1.

Next, to test the 11·86-year period, the same numbers were arranged in groups of twelve. Owing to the fact that 120 years is rather more than ten periods, to save trouble the device adopted in the reduction of tidal observations was used —an extra number, the mean of those immediately preceding and following, was inserted in the middle of the series, which thus extends over 119 years. The resulting twelve means, each formed from ten annual means, are as follows :—

1	2	3	4	5	6	7	8	9	10	11	12
25	33	47	57	73	72	65	60	55	43	29	20

As before, the numbers 1, 2 . . . 12 are parts of a period; the values have been plotted in fig. 2.

11·86-year Period.

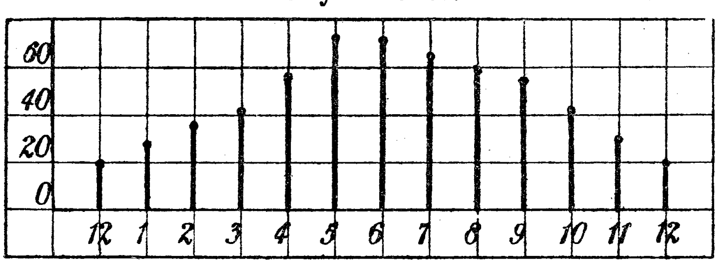


FIG. 2.

In each case the differences from the value of the mean, 48, seem to justify an assertion as to the existence of the periods.

An attempt was next made to see if any of the other periods were to be found in the Sun-spot numbers. The following are those which seemed most likely to be shown on account of the magnitudes of the forces :—

Mercury, effect of eccentricity, ·24085 year.

M—V, ·19795 year,	V—J, ·32242 year,
M—E, ·15863 „	E—J, ·54603 „
M—J, ·12292 „	V—E, ·79936 „

where M, V, E, J, denote *Mercury*, *Venus*, the Earth, and *Jupiter*, respectively, and M—V, M—E, . . . , the halves of their relative periods. The halves of all the periods, except that due to the eccentricity of *Mercury*, must be used owing to the fact that two equal waves are raised on opposite sides of the Sun, and we are only concerned here with the coincidences of these waves. All of these periods are short, and it is very doubtful whether the observations can be taken with sufficient accuracy to test any of them. However, an attempt was made with the longest, V—E, which has a period of almost exactly four-fifths of a year. In the course of sixty years the deviation from an exact period of four-fifths would be less than a fortnight. I therefore took Wolf's monthly means from 1831–1890, which is almost exactly a long-period cycle, and arranged them in 48 columns corresponding to the 48 months in a four-year period. These 48 means, each formed from 15 numbers, should give 5 maxima and 5 minima if the action of V—E is to be shown. The means are given in the following table :—

Month / Year.	1	2	3	4	5	6	7	8	9	10	11	12
1	42·3	46·5	50·4	47·4	44·3	41·9	43·3	54·7	52·7	57·6	53·2	47·9
2	54·5	57·3	55·6	52·3	51·7	52·2	53·2	51·7	50·2	55·4	52·2	59·4
3	54·9	58·1	51·5	51·7	51·2	54·7	51·3	49·5	46·0	43·9	43·1	48·1
4	43·8	46·9	53·7	49·2	50·9	45·3	44·1	43·7	51·3	46·8	45·3	40·9

Plotting these as before, we obtain the curve in fig. 3, where the numbers 18, 22 . . . show the ordinates which correspond to year 2, month 6; year 2, month 10, etc., in the above table.

4-year Period.

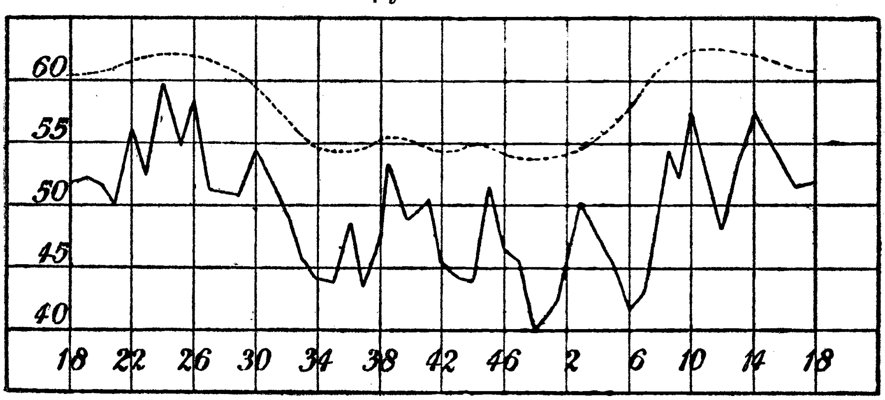


Fig. 3.

The four-year period which appears to result from this curve is illusory. If a succession of 12-year periods be plotted in groups of four years, the maxima will all fall in one place; thus the apparent four year period is due to the 11·86-year period. Similarly, the apparent approximate symmetry about the centre of the curve in fig. 4, as shown by the dotted line, is due to the 9·93-year period; an even number of such periods arranged in groups of 4 years will give an approximately symmetrical curve.

As we are searching for a $\frac{4}{5}$-year period, the numbers in the table should contain five periods. Dividing each period into nine equal parts and estimating their magnitudes from fig. 4,* we obtain the following series of means for the nine parts of the period :—

1	2	3	4	5	6	7	8	9
50·7	49·9	51·0	50·0	48·5	49·0	49·6	49·4	49·9

which are plotted in fig. 4.

·7994-year Period.

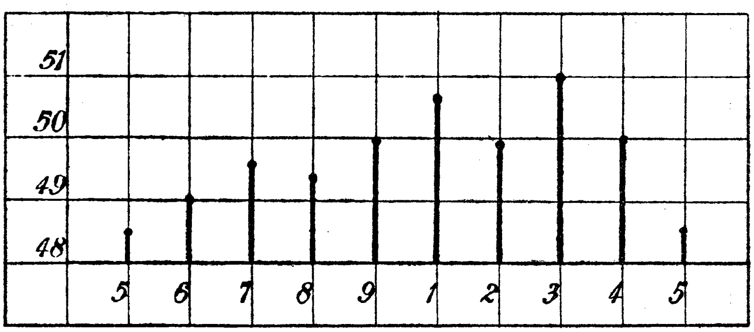


Fig. 4.

Although each of the nine means arises from about 80 monthly means, and although their general change seems to afford some evidence of the required period, the differences from their mean value are too small to give more than a doubtful indication of the existence of the period.

1900 Aug. 4.

* They were actually estimated from a large-scale figure.